PHYSICAL REVIEW E **77**, 060301(R) (2008)

Characterization of reflection intermittency in a composite granular chain

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(Received 2 December 2007; published 23 June 2008)

The physical factors controlling the power-law behavior of impact energy in a composite granular chain remain elusive. Based on event-driven simulations and the on-off intermittency of wave reflections, we obtain the probability distribution functions of the waiting time τ and the energy leakage ΔE. They exhibit lognormal distributions, which together with the relationship between ΔE and τ allow one to explain directly the power-law behavior of the confined energy. This work may be extended to higher dimensions and help us understand the complex dynamics in granular materials.

DOI: 10.1103/PhysRevE.77.060301 PACS number(s): 45.70.−n, 43.25.+y, 46.40.Cd, 47.20.Ky

Granular materials have been attracting significant interest because of their ubiquity in nature and industrial processes, but it is difficult to understand their intrinsic dynamic properties due to the strong nonlinearity of forces between particles and the complex distributions. A one-dimensional granular chain as a simple starting point of studying higher-dimension granular materials gives rise to rich phenomena [1–5]. As reported in the pioneering work of Nesterenko [1], the propagation of an elastic impulse in a granular chain possesses solitonlike features. Recently, Hong [6] constructed a "granular container" using a series of sections with particles and predicted that this granular container can trap energy in a particular region and release the trapped energy little by little over time. He also found that a universal power-law behavior of the impulse energy remained inside various granular containers: $E_R = At^{-\gamma}$, where the scaling exponent γ is a universal dimensionless constant. Daraio *et al.* [7] demonstrated experimentally the efficiency of solitonlike and shocklike pulse trapping and disintegration in the granular protector. Doney and Sen [8] found the impressive mitigation capability of decorated and tapered granular chains increases with the number of spheres and tapering. In a recent numerical study [9] we observed a marked crossover in the power-law behavior of the impact-energy decay, which is linked to the structural transition from the compression to dilation state in both heavy-particle sections. The average reflection frequency first increases and arrives at its maximum at "crossing" time, and then decays almost exponentially. Yet, the origin of the appearance of power-law behaviors and an apparent crossover remains to be clarified.

As is known recently, (i) when the solitary wave passes from heavier particles into lighter ones, it breaks into a train of weaker and slower pulses. (ii) When a pulse moves from lighter particles to heavier ones, a significant part of it is reflected back through the interface. (iii) In the process of solitary wave collisions secondary solitary waves are generated. So a composite symmetrical granular chain (containing one light section in the middle and two heavy sections at the two ends) is filled with a large number of pulses bouncing back and forth between two interfaces and trickling out the trapped energy slowly. The common feature of the large number of solitary waves may be their on-off intermittent dynamics: any solitary wave moves freely between two interfaces with no energy dissipation (this process is defined as the "off" state) and bounces back at the interfaces accompanying the energy leakage (defined as the "on" state), resulting in the intermittent reflections associated with the intermittent energy leakage. The off state is quiescent and remains without energy dissipation for long periods, while the on state is a burst of energy leakage, departing from and returning to the off state quickly. The collective behavior of large numbers of intermittent processes may be independent of the details of individual attributes and instead be the consequence of the generic property. Nonetheless, to the best of our knowledge, the study of a composite granular chain has not been reported in terms of the on-off intermittent mechanism. The stochastic form of intermittency of the chaotic system has been observed in many mathematical models and experimental paradigms [10]. If the time interval between successive reflections of a pulse is called the waiting time (τ), these time intervals could be referred to as the laminar phase. The intermittency may be characterized by the probability distribution functions (PDFs) of the waiting time and corresponding energy leakage. The most common nature of composite granular chains may be their intermittent dynamics and the power-law behavior may be understood in terms of the PDFs of the waiting time and the energy leakage, which is the focus of this Rapid Communication.

Here we modeled the symmetrical granular chain containing two sets of hard spheres and divided into two heavy (120+120 particles) and one light sections (180 particles) and hence having two interfaces [see Fig. 1(a)]. The heavy (light) particles have masses m_1 (m_2) and radii R_1 (R_2). Both ends of the chain are free to move. A simple event-driven method [11] well suited to describe hard spheres is used. No dissipation on collisions is taken into account. Event-driven simulations make it easy to track each individual particle in time and each reflection of a pulse at the interfaces and thus to conveniently obtain the information on waiting time and energy leakage. The diameter of hard spheres is 100. The governing parameter in the hard-sphere chains is mass, so we modify the properties of granular chain through changing the

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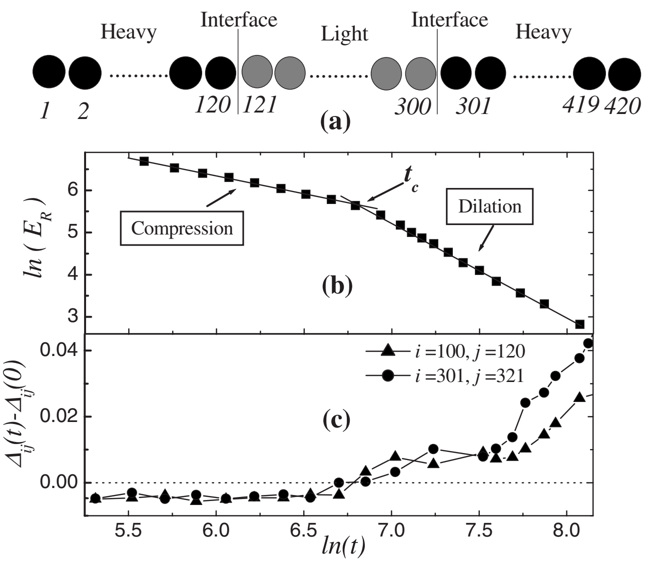

FIG. 1. (a) Schematic setup of the symmetric hard-particle chain containing two sets of particles. (b) The time dependence of the impact energy remained inside the light-section particles. Two solid lines are guides to the eyes and correspond to the compression and dilation branches, respectively; they intersect at time t_c. (c) The compression and/or dilation parameter $\Delta_{ij}(t)-\Delta_{ij}(0)$ as a function of the time for heavy particles nearby the interfaces.

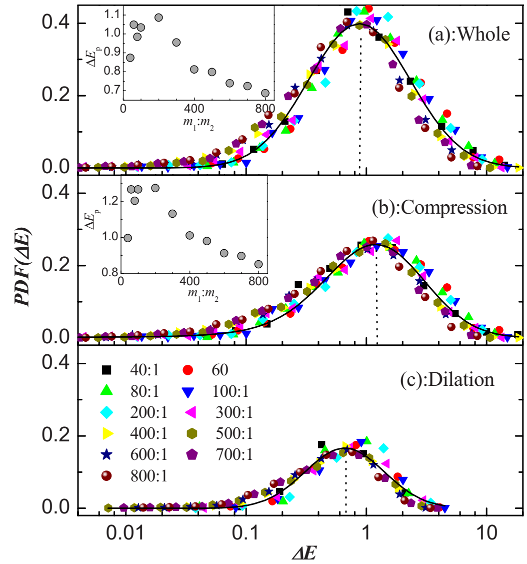

FIG. 2. (Color online) Probability distribution functions (PDFs) of energy leakage (ΔE) at the interfaces in the whole energy-decay process (a), in compression (b) and dilation (c) regions. Different m_1 values are indicated in the figure. Solid lines are lognormal distribution fits. Insets in (a) and (b) show the obtained peak positions ΔE_p against the mass ratio $m_1{:}m_2$.

mass ratio, and for convenience m_2 is set to be 1. An impulse defined by an initial velocity ($V=10$) at time $t=0$ was initiated at the first particle. The parameter Δ_{ij}, introduced as in previous work [9], represents the degree of compression or dilation of the granular chain between particles i and j. The chain between particle i and j is in the dilation state if $\Delta_{ij}>0$ and in compression state if $\Delta_{ij}<0$. The initial state corresponds to $\Delta_{ij}(0)=0.01$ for any i and j.

Our first observation is that a limited part of the system is crossing the initial state at the crossing time t_c. As displayed in Fig. 1(b), we observe the same crossover in the power-law behavior of confined energy as that found in the soft-sphere chain [9]: the leakage of confined energy contains two scaling regions with different exponents, denoted by the compression and dilation branch, respectively. Figure 1(c) shows $\Delta_{ij}(t)-\Delta_{ij}(0)$ as a function of time for heavy particles near the interfaces. Note that here the vertical axis title is $\Delta_{ij}(t)-\Delta_{ij}(0)$, indicating the degree of compression or dilation *relative to the initial state*. This figure clearly reveals that the difference $\Delta_{ij}(t)-\Delta_{ij}(0)$ increases from negative to positive with the time, and the time when its sign changes is in good agreement with t_c shown in Fig. 1(b), namely, $\Delta_{ij}(t_c)=\Delta_{ij}(0)$. Hence the observed crossover in the power-law behavior is linked to the structural transition in both heavy sections from the compression to dilation state *relative to the initial state*. The result, $\Delta_{ij}(t_c)=\Delta_{ij}(0)$, is further confirmed in the hard-sphere chain with $\Delta_{ij}(0)=0.1$ and the soft-sphere chains with a precompression (induced by a constant force $F=1$) and a predilation $[\Delta_{ij}(0)=0.01]$ by the molecular dynamic method. $\Delta_{ij}(t_c)=\Delta_{ij}(0)$ for the heavy particles close to both interfaces reveals that at transition point and/or region a part of two heavy sections is crossing its initial state.

Now we focus on the features of the energy leakage and the waiting time. ΔE represents the energy leakage from the light section to the heavy section after a reflection. Because the obtained ΔE values span a wide range of more than ten decades, it is found that the $\mathcal{P}(\Delta E)$ calculated using the logarithmically scaled size of boxes $[\mathcal{P}(\Delta E)=\frac{1}{N}\sum_i\delta(\log_{10}\Delta E_i-\log_{10}\Delta E)]$ is smoother than that usually obtained using the linearly scaled boxes $[\mathcal{P}(\Delta E)=\frac{1}{N}\sum_i\delta(\Delta E_i-\Delta E)]$. A comparison between the $\mathcal{P}(\Delta E)$'s for different granular chains is presented in the semilogarithmic scale in Fig. 2(a) and reveals a noticeable result: at first glance the $\mathcal{P}(\Delta E)$'s collapse onto the same curve for all granular chains, indicating that there may exist the universality in $\mathcal{P}(\Delta E)$'s in the whole energy-leakage process. So all data are fitted with a lognormal function, $f(\Delta E)=\frac{a}{\sqrt{2\pi}\Delta E b}e^{-[\ln(\Delta E)-c]^2/2b^2}$, shown by the solid line in Fig. 2. Note that the lognormal distributions are skewed, and are symmetrical at the logarithmic level. We obtain the standard deviation $b=0.98$ and the expected or mean value $c=0.83$. The peak at $\Delta E_p=0.88$ marks the predominant interval, which increases linearly with the square of impact velocity (V^2) as expected. We further evaluate these data with different lognormal functions for different mass ratios, respectively. The obtained peak positions (ΔE_p) are presented against the mass ratio in the inset of Fig. 2(a). It is clear that the mean value of ΔE depends on the mass ratio: ΔE_p almost decreases exponentially with mass ratio except for the big deviation at mass ratio 40:1. The standard deviation and the peak width are insensitive to the mass ratio (not shown here). Similarly, the $\mathcal{P}(\Delta E)$'s in compression and dilation regions are presented in Figs. 2(b) and 2(c), respectively. Whether in the compression region or in the dilation region the $\mathcal{P}(\Delta E)$'s could also be fitted roughly with a "universal" lognormal function, respectively: in the compression region, $b=0.90$

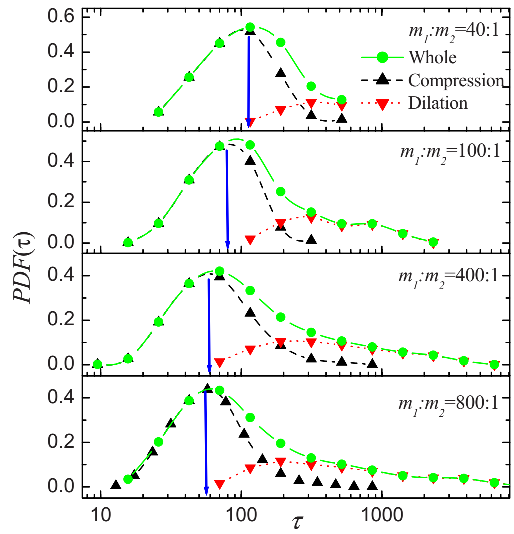

FIG. 3. (Color online) Probability distribution functions (PDFs) of the waiting time (τ) in the whole energy-decay region, in compression and dilation regions for four various mass ratios as indicated in the figure. The arrows denote the positions (τ_p) of PDF peaks in the compression region.

and the peak locates at 1.25; in the dilation state $b=0.76$ and the peak is at 0.67. In the same way above, the further quantitative analysis in the compression region has been made of the mass ratio effect on the peak position and is presented in the inset of Fig. 2(b). A similar conclusion could be made: ΔE_p almost decreases exponentially with mass ratio except for mass ratio 40:1, whereas the standard deviation and the peak width are not sensitive to the mass ratio. Here the qualitative analysis of this mass-ratio effect is made as follows. Applying energy and momentum conservation, at the heavy-light interface when a heavy particle with velocity V_h collides with a stationary light particle, the velocities become after the collision $V_h'=\frac{m_1-m_2}{m_1+m_2}V_h$ and $V_l'=\frac{2m_1}{m_1+m_2}V_h$; at the light-heavy interface when a light particle with V_l' collides with a stationary heavy particle, the velocities become after the collision $V_l''=\frac{m_2-m_1}{m_1+m_2}V_l'$ and $V_h''=\frac{2m_2}{m_1+m_2}V_l'$. So, for the first subwave, during its first reflection $\Delta E_{11}=\frac{1}{2}m_1(V_h'')^2=\frac{8m_1^3m_2^2}{(m_1+m_2)^4}(V_h)^2$[in the next reflection $\Delta E_{12}=(\frac{m_1-m_2}{m_1+m_2})^2\Delta E_{11}$ and so on]. For the second subwave, $\Delta E_{21}=\frac{8m_1^3m_2^2}{(m_1+m_2)^4}(V_h')^2$, etc. Therefore, when $\frac{m_1}{m_2}\gg 1$ ΔE_p decreases exponentially with mass ratio, which is in agreement with our results; the big deviation for mass ratio 40:1 should be related to the insufficient statistics.

The $\mathcal{P}(\tau)$'s are shown in the semilogarithmic plot in Fig. 3 (for clarity only four different mass ratios are presented). Compared to the $\mathcal{P}(\Delta E)$'s the $\mathcal{P}(\tau)$'s are quite smooth whether they are calculated using logarithmically or linearly scaled size of boxes. For $m_1:m_2=40:1$ shown in Fig. 3(a), the data of long waiting time are relatively less since the smaller the mass ratio the faster the energy decay. The whole $\mathcal{P}(\tau)$ consists of a peak in the short waiting-time side and a very broad hump in the long waiting-time side. The peak arises from the reflection intermittency in the compression state and the broad hump stems from the reflection intermittency in the dilation state. The peak is nearly symmetrical suggesting that $\mathcal{P}(\tau)$ can be fitted by a lognormal distribution function and its width is not sensitive to the mass ratio, whereas its position (τ_p, denoted by the arrows) moves toward short waiting time with the mass ratio. Hence, the mean value of τ in the compression state decreases with the mass ratio. On the one hand, the free-end boundary condition is used in our present case, so the dilation state exists, where the gaps between grains appear in heavy sections and become wider with time. On the other hand, after the energy-decay processes in the compression state, in the dilation state not only do the pulses become much weaker but also there is a big decrease in the number of pulses. Therefore a very broad hump located at the long-time side of the PDF curve is observed in the dilation state, in contrast with the symmetry peak located at the short-time side in the compression state, and it may be inappropriate to study the dilation state in connection with the on-off intermittency despite the fact that we calculated the PDF of the waiting time τ.

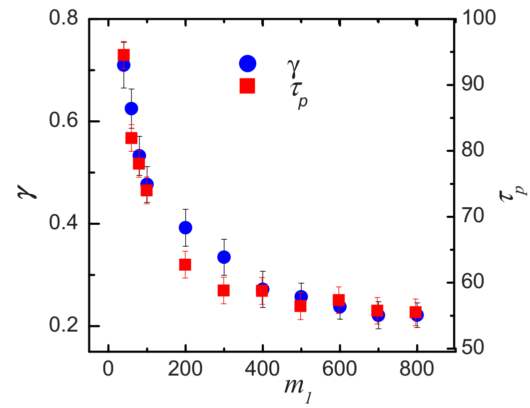

FIG. 4. (Color online) A comparison of the mass-ratio dependence of the power-law exponent (γ) in the compression state and the peak position (τ_p) of the $\mathcal{P}(\tau)$ in the compression region.

According to the above results and discussions, the power-law behaviors of the confined energy could be well understood. Two factors control the energy decay: the energy leakage and the waiting time. The former follows lognormal distributions in the whole impact-energy-decay process. The latter follows lognormal distributions in the compression state and a likely broad distribution in the dilation state. In the compression state the expected value of τ increases with mass ratio. As aforementioned, the off state is quiescent and remains without energy dissipation for long periods τ, while the on state is a transient energy release ΔE. Because of the energy leakage in the on state, high-energy pulses decay to low-energy pulses and low-energy pulses decay to lower-energy ones, implying the pulses decay in a cascadelike manner, and the structures of laminar phase τ break up into wider and wider structures. These suggest the appearance of the "law of proportional effect." For a high-energy pulse, ΔE is large and the corresponding τ is short, while for a low-energy pulse ΔE is small and the corresponding τ is long. Hence, with the time ΔE values become smaller and smaller and τ values become larger and larger, between them there exists a power-law relation with a negative exponent, which is confirmed by our data. Meantime, the cascadelike manner and the law of proportional effect provide the impetus to understand lognormal distribution of ΔE and τ [12]. Both governing factors follow the lognormal probability distributions in the compression state. So, we can understand the power-law decay of the confined energy. The great difference in the $\mathcal{P}(\tau)$'s between compression and dilation states leads to the marked crossover in power-law behavior. In addition, for the compression state the scaling exponent (γ) and the expected value of the waiting time (here approximated by τ_p, see Fig. 3) as a function of mass ratio are presented in Fig. 4. Two sets of data overlap roughly regardless of their difference in units, indicating that the mass-ratio dependence of τ plays a leading role in the mass-ratio dependence of γ.

In conclusion, based on the on-off intermittency of solitary-wave reflections we obtain the $\mathcal{P}(\tau)$ and $\mathcal{P}(\Delta E)$. Both $\mathcal{P}(\tau)$ and $\mathcal{P}(\Delta E)$ follow lognormal distributions and there exists an important difference in the $\mathcal{P}(\tau)$ between compression and dilation states, which together with the relationship between ΔE and τ make comprehensible the power-law behavior of the confined energy and the difference in the scaling exponents between compression and dilation as well as the mass-ratio dependence of power-law exponents. Intermittency is a prominent phenomenon observed in a large variety of nonlinear dynamical behavior. The statistical properties of solitary waves studied in this work are fundamentals of the dynamics in composite granular chains. This work may be extended to higher dimensions, e.g., the features of successive-collision intervals in granular flows and in a granular mixture subject to vibrations. In addition, it would be interesting to make a detailed comparison of the present case with the three types of intermittency introduced by Pomeau and Manneville, as well as crisis-induced intermittency [13].

We thank Z. R. Yang, Q. F. Fang, G. W. Meng, and Y. Q. Wang for useful discussions. This work was supported by the National Natural Science Foundation of China (Grant No. 10674135) and by the Center for Computational Science, Hefei Institutes of Physical Sciences.

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